Working with composite indicators to measure environmental sustainability

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Abstract

The construction of composite indicators often involves the aggregation of several indicators into a single measure. These individual indicators typically represent different aspects of the phenomenon being investigated, such as economic, social, or environmental factors. Since these indicators are measured in different units, scales, and ranges, a critical step in the construction is the normalization step. Normalization transforms data into a common scale enabling meaningful comparison and allowing for aggregation. However, normalization introduces several challenges and potential pitfalls that can affect the validity and interpretability of composite indicators. Here, we focus on the max-min normalization method, and we modify it by allowing to vary the classical minimum and maximum values in a given interval, respectively. Adjusting these values within a range generates a distribution of normalized values for each indicator. Consequently, we can take the mean value and the corresponding error of measurement as normalized value of the indicator. The error can be interpreted as a sort of degree of reliability associated with the mean value. Finally, we aggregate those values along with their errors of measurement. An empirical application, based on environmental data for Italian Provinces, illustrates how the method works.

Keywords: Composite indicators, Normalization, max-min method, environmental sustainability.

1 Introduction

In the construction of a *composite indicator*, researchers have to face a common practical problem, namely how to make comparable indicators. This is the well-known normalization step. While on the one hand, many articles have been written about the construction of indicators, the preliminary normalization step has received less attention. However, the choice of a normalization method is not harmless since different methods yield different results, which can influence outcomes ([1]). Literature accounts for several normalization methods, such as: i) *Ranking of indicators across countries*[1]; ii) *Standardization (or z-*

[1]This is the simplest normalization approach consisting in ranking each indicator across countries. Despite its simplicity and resilience to outliers, the method has a significant drawback: it results in the loss of information about absolute levels and makes it impossible to draw conclusions about differences in performance.

scores)[2]; iii) *Re-scaling*[3]; iv) *Distance to a reference country*[4]; v) *Categorical scales*[5]; vi) Indicators above or below the mean[6] (see [2] for a detailed review on normalization methods.).

The approach we propose defines as normalized value for a given unit the mean of the distribution of normalized indicators with a measurement error that represent the uncertainty around this value. In this way, each unit is characterized by a value (the mean) and its own confidence interval, and two units are considered significantly different if their intervals do not overlap.

The method is used to represent uncertainty or variability in the data, providing a range of possible values rather than a single point estimate (see, for example [3]). Intervals for composite indicators also appear in [4] but in that paper authors fix a normalization and then use a Monte Carlo simulation to account for several possible systems of weights in the aggregation step.

The method we propose has several advantages. Firstly, it introduces robustness into the ranking since two units are really different if their intervals do not intersect. Secondly, the expansion of the possible ranges for the minimum and maximum value allows for temporal comparisons. In fact, with this approach, when a new year is added, it may not be necessary to perform a new normalization since the new values of maximum and minimum could belong to the ranges mentioned above. Finally, from a technical point of view, the normalized values fall in the range $(0, 1)$ and, since it avoids zero values, in the aggregation phase we can apply any generalized mean [5], which in general only works for strictly positive values.

In recent years, measuring environmental sustainability has become increasingly important due to the growing challenges regarding climate change, biodiversity loss, and resource

[2]Normalized values are obtained by subtracting the average value across countries from the raw value and dividing by the standard deviation. This standardization is widely used because it converts all indicators to a common scale (average = 0, standard deviation = 1), preventing aggregation distortions caused by differing means.

[3]Normalized values are calculated as the ratio of the difference between the raw value and the minimum value, divided by the range, as consequence, normalized values range in $[0, 1]$. It is also known as *max-min method*. This approach is sensitive to unreliable outliers, as minima and maxima can distort results. It also amplifies the influence of indicators with small value intervals on the composite indicator. For time-dependent studies, the range is often fixed across the entire time frame. If new data exceed the selected range, the composite indicator must be recalculated for all years to ensure comparability.

[4]This method normalizes an indicator by dividing its value for a given country at a specific point in time by the value of a reference country at an initial time. This approach accounts for the evolution of indicators over time.

[5]To get normalized values, each indicator is assigned a categorical score, which may be numerical or qualitative (e.g., "fully achieved," "partly achieved," "not achieved"). In some cases, the scores are determined based on the percentiles of the indicator's distribution across countries. Categorical scales offer the advantage that small changes in the indicator value (e.g., over time) do not impact the normalized value. However, this can also be a drawback, as a significant amount of information about the variance between countries in the normalized indicators is lost.

[6]This normalization method differentiates between values that are above, near, or below an arbitrarily defined percentage threshold around the mean. The normalized value is set to 1 if the indicator exceeds the threshold, -1 if it falls below, and 0 if it lies within the neutral zone around the mean. The method's simplicity and robustness against outliers are its key advantages, while its drawbacks include the arbitrary nature of the threshold and the loss of information about absolute levels.

depletion. Thereby, measuring environmental sustainability at local level, such as Regions (NUTS2) or Provinces (NUTS3) is crucial for tailoring local policies for specific ecological heeds and understanding of regional differences in air quality, water usage, energy consumption, and waste management. Focusing on provincial-level data allows policymakers to design more effective interventions than national-level strategies while ensuring better resource allocation.

Here, we apply our approach to measure environmental sustainability of the Italian Provinces using the *Sole24ore* data collected in 2024.

The rest of the paper is organized as follows. Section 2 introduces the method and Section 3 illustrates its potential by analyzing environmental data. Section 4 concludes.

2 The method

Let X be a $n \times k$ matrix whose element x_{ij} represents the value of the j-th elementary indicator $j = 1, \ldots, k$ for the i-th local unit (e.g., the Italian province) $i = 1, \ldots, n$. We denote by r_{ij} the normalized value, obtained according to the classical *min-max method*:

$$r_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}. \tag{1}$$

where, $\min_i x_{ij}$ and $\max_i x_{ij}$ denote the minimum and maximum value across all units, respectively

As stressed in Section 1, the choice of maximum and minimum can lead to different results. Moreover, adding a new year may require to update values from previous years. For this reason, we modify the method as follows: we fix a *maximum*, an *upper bound*, UB and a *minimum*, a *lower bound*, LB external to the intervals and proportional to the real $\max_i x_{ij}$ and $\min_i x_{ij}$ for each variable j.

Then, we normalize the data by choosing as maximum and minimum value in (1) any possible number in the interval $[\max_i x_{ij}, UB(x_{ij})]$ and $[LB(x_{ij}), \min_i x_{ij}]$, respectively. More in details, for the maximum (minimum), we move from $\max_i x_{ij}$ $(LB(x_{ij}))$ to UB (LB) by ϵ_U $(\epsilon_L) > 0$, that is

$$\epsilon_U = \frac{UB(x_{ij}) - \max_i x_{ij}}{\delta_U} \qquad \epsilon_L = \frac{\min_i x_{ij} - LB(x_{ij})}{\delta_L} \tag{2}$$

where δ_L, δ_U are the number of parts into which we want to divide the interval and without loss of generality, we can fix $\delta = \delta_L = \delta_U$, with $\delta = 100.$ in our empirical analysis. Consequently, equation (1) can be re-written as follows:

$$Z_{ij}^v = \frac{x_{ij} - a_v}{b_v - a_v} = \frac{x_{ij} - (LB + v \cdot \epsilon_L)}{(UB - v \cdot \epsilon_U) - (LB + v \cdot \epsilon_L)} = \frac{x_{ij} - LB - v \cdot \epsilon_L}{(UB - LB) - v \cdot (\epsilon_U + \epsilon_L)} \; v = 0, \ldots, \delta \tag{3}$$

As a consequence, for each v, $v = 0, \ldots, \delta$ we get a (column-)vector of normalized values starting from x_{ij} and we collect these vectors in a matrix of size $n \times \delta$. That is, we have a matrix for each variable and, consequently, k matrices of the same size, one for each indicator

j, $j = \ldots, k$. We interpret each column of the matrix as a realization of the $j-th$ variable over the units. That is, we have a random vector of normalized variable Z_j and we dispose of δ realization of this random vector.

To clarify this point, suppose we have an indicator that varies within the range $[10, 90]$. To normalize, we set $\delta = 11$ and $\epsilon = 1$, consequently $v = 0, 1, \ldots, 10$. Thus, we can normalize the data by choosing numbers from 90 to 100 as the maximum and from 0 to 10 as the minimum. The values 0 and 100 are LB and UB, respectively. So, by choosing $v = 0$, we have $min = 10$ and $max = 90$, which corresponds to the classic min-max. When $v = 1$, $min = 9$ and $max = 91$. Therefore, we obtain a different normalized vector. We, thus, repeat the normalization process, generating 11 normalized vectors.

2.1 A simple proposal

As first proposal, we decide to define the *normalized value* for unit i as the mean value of the distribution of normalized indicators, $\overline{Z}_j$, and as measurement error the marginal error of the $(1-\alpha)\%$ confidence interval for the mean value, ΔZ_j. That is,

$$\overline{Z}_1 \pm \Delta Z_1, \quad \overline{Z}_2 \pm \Delta Z_2, \quad \ldots, \quad \overline{Z}_k \pm \Delta Z_k \qquad j = 1, \ldots, k. \tag{4}$$

We observe that ΔZ_j is the half-width of the confidence interval, that is the marginal error.

In the simplest case, we assume that, the composite indicator for each unit i is the arithmetic mean among the $\overline{Z}_j$ $(j = 1, \ldots, k)$ indicators. We denote this quantity by $M = \frac{1}{k}\sum_{j=1}^{k} \overline{Z}_j$. However, as displayed in (4), each measure has an own confidence interval and we can take them into account. In this way we can affirm that two units are really different if their confidence intervals do not intersect.

To achieve this aim, we apply the classical physics law of propagation of uncertainty [6], working under the assumption og Independent Errors. More in detail, when the errors are independent, the uncertainty of the mean is: $\Delta M = \frac{1}{\sqrt{k}}\sqrt{\sum_{j=1}^{k}(\Delta Z_j)^2}$. Thus, the final result is:

$$M \pm \Delta M = \left(\frac{1}{k}\sum_{j=1}^{k} \overline{Z}_j\right) \pm \frac{1}{\sqrt{k}}\sqrt{\sum_{j=1}^{k}(\Delta Z_j)^2} \tag{5}$$

3 An illustrative example

3.1 Data

Data belong to the 35th edition (December, 2024) of the *Sole 24 Ore Quality of Life*.[7].

The Sole 24 Ore Quality of Life is an annual ranking of Italian provinces based on several indicators to capture the quality of life in those regions. It is published since 1990 by the Italian financial newspaper *Il Sole 24 Ore* and it is widely recognized as a comprehensive tool for assessing and comparing living conditions across the 107 Italian provinces.

[7]Data collected by the *Sole24ore* are available on the Sole Ore GitHub page by citizens, researchers, media and decision makers.`https://github.com/IlSole24Ore`

The ranking is based on six macro-categories and 90 indicators that capture different dimensions of well-being: i) Wealth and Consumption (it includes indicators such as per capita income, household savings, and property prices); ii) Business and Labor (key indicators include unemployment rates, the number of new businesses, and productivity levels); iii) Demography and Society (it includes population growth, aging, and migration rates); iv) Environment and Services (examples include air pollution levels, green spaces, public transportation, and waste management); v) Justice and Security (such as crime rates, road safety, and the efficiency of the judicial system), vi) Culture and Leisure (it includes the number of theaters, museums, sports facilities, and entertainment options.

Among the 90 indicators we focus on 3 variables related to environmental sustainability. More in details, we use 1) Urban ecosystem (Ecourb, Synthetic index on 18 parameters); 2) Electricity from renewable sources (Enrin, Incidence of wind, photovoltaic, geothermal and hydro, in % of gross production); 3) Protected Areas (Areeprot, %).

Table 1: Summary Statistics

	$\overline{Z}_1$(Ecourb)	$\overline{Z}_2$(Enrin)	$\overline{Z}_2$(Areeprot)	M	LB(M)	UB(m)
Min.	0.01152	0.001802	0.002747	0.1926	0.1902	0.1950
Median	0.59423	0.495387	0.273077	0.4482	0.4438	0.4526
Mean	0.58248	0.501206	0.315993	0.4666	0.4629	0.4703
Max.	0.94300	0.951535	0.950706	0.8304	0.8254	0.8353

Notes. Our computation on Sole24ore data.

Figure 1 reports the 20 Provinces that occupy the Top position (left panel) and the Bottom positions (right panel). Blu line refers to the average value in the subset of Provinces. Belluno and Verbano-Cusio-Ossola occupy the first and second position, respectively. Their values are (statistically) different whereas Salerno and Sondrio can be ranked at the same position since their intervals intersect. In the bottom of the ranking we can find Brindisi whixh displays the worst performance whereas there is no differences among Catanzaro, Lodi, Milano, Sicarusa and Napoli.

4 Conclusions

This work is a very preliminary attempt to account reliability on the normalization step by proposing to construct a distribution of normalized values. Future research will be directed towards involve error measurement according to different aggregation methods and to propose test on ranking differences.

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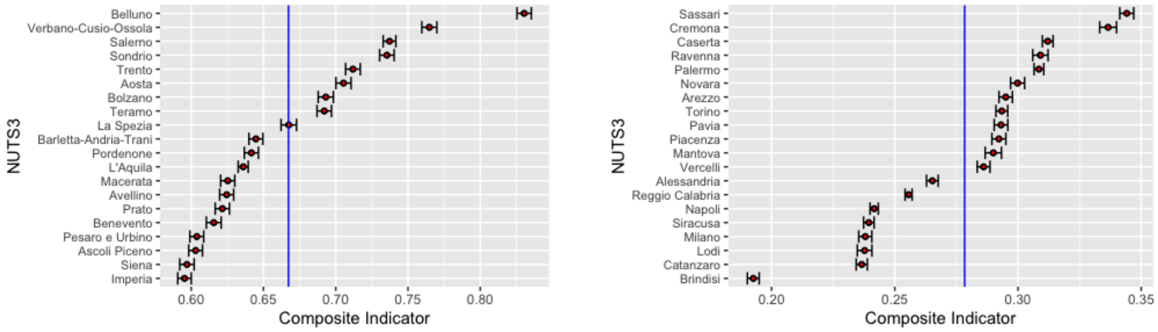
Figure 1: Composite indicators with measurement errors



(a) Top 20 Provinces

(b) Bottom 20 Provinces

Mission 4, Component 2, Investment 1.1, CUP I53D23007340001.

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